CHAPTER ONE

THE FUNDAMENTAL PHYSICAL QUANTITIES

In order to study the science of music, it is necessary to learn some of the technical vocabulary of acoustics. This requires that we first define a number of technical acoustical terms so that we may use them properly and without ambiguity. However, acoustics is the study of systems that produce and propagate what we recognize as sound, and is based on the larger area of science called physics. We must therefore begin by learning some of the technical vocabulary of physics. The entire vocabulary of physics is quite extensive, but we will need to concern ourselves with only a small part of it; that part, however, will be quite indispensable.

The science of physics begins by considering objects and concepts with which we are intuitively familiar because we deal with them constantly in our everyday experience. However, the discipline of physics refines our thinking about these things not only by defining them as rigorously as possible, but also by making these definitions quantitative. This makes it possible to describe our objects and concepts by using numbers; they then become what we will call *physical quantities*, and we can discuss them with much more precision than we can discuss things to which we cannot attach numbers.

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When dealing with physical quantities as described by numbers, the use of elementary mathematics becomes not only useful but indispensable. Mathematics is an abstract construction of the human mind and it is really quite miraculous that it should have an immediate and practical application to the real world, serving as a quantitative language with which to discuss those things that can be described by numbers. Mathematics, like physics, covers an extensive area, but what we will need is even a smaller part than we need of physics; the barest essentials of algebra will be sufficient. Those who view mathematics as beyond understanding may be comforted by the assurance that the little we will employ will require not quite as much effort as balancing a checking account at the end of the month. What we will need is not so much the paraphernalia of mathematics as the practice it develops of thinking in quantitative terms. To try to dispense completely with mathematics and mathematical notation would be comparable to trying to study harmony without using the musical staff or musical notation.

With these preliminaries out of the way, we may begin our discussion of physics. We start by defining three fundamental physical quantities: length, time, and mass. When these are defined, other physical quantities can then be defined in terms of them. The quantities chosen as fundamental are actually selected rather arbitrarily and could be replaced by others; however, the three that have been selected serve as well as any.

Length

The first fundamental unit chosen is that of *length*. This is a physical quantity with which we are all quite familiar. It is associated with the equally familiar concept we call *distance*, the spatial separation of two points. To determine a distance, we first select some agreed unit of length and see how many times this unit is contained in the given distance. The result of this process is called a *measurement*. In general, our unit of length will fit into the given distance a certain whole number of times, with something left over. By subdividing the unit of length into smaller portions, the amount left over can be measured in terms of these subdivisions. With sufficiently fine subdivisions, the measurement of distance can theoretically be made as accurate as we please.

To be widely useful, the unit of length should be one agreed upon by the majority of users; it then becomes a *standard of length*. Our present standard is the *meter*. It was originally meant to be one ten-millionth or 10^{-7} of the distance along the surface of the earth from the north pole to the equator. (The exponential notation used here and subsequently is explained in the Appendix.) This original intention turned out to be impractical, so the meter was subsequently defined arbitrarily as the distance between two fine scratches on a particular metal bar constructed for the purpose. This standard is kept at the International Bureau of Weights and Measures in Sèvres, France; copies of it are distributed to other countries to serve as subsidiary standards. (In 1960 this standard had become too inaccurate for present-day measurements, and the meter was redefined in other terms; however, we need not concern ourselves further with it.) The system of units based on the meter as the standard of length is called the *metric system*.

The meter serves as the standard of length all over the world. This is true even in England and the United States, which do not customarily use the meter as a unit of measurement, but instead use the *foot*. However, the foot is not based on a separate standard; it is defined in terms of the meter, the relationship being 1 foot = 0.3048 meter, exactly. The foot is the unit of length in the English system.

For the measurement of lengths it is convenient to have available various-sized multiples and sub-multiples of the standard. Those in the English system are quite inconveniently arranged, with 12 inches = 1 foot, 3 feet = 1 yard, 5280 feet = 1 mile, and so forth. This creates unnecessary trouble; for example, to convert a distance given in feet to the same distance expressed in miles, we must divide the number of feet by the awkward number 5280.

The metric system is much more sensibly arranged in this respect. All multiples and sub-multiples are expressed in terms of powers of 10, such as 100, 1,000, and so on. A length of 0.01 meter is called a *centimeter*, and 1,000 meters constitutes one *kilometer*. Conversions are now much simpler. To change a distance given in meters to the same distance expressed in centimeters, we multiply the number of meters by 100, and this merely means moving the decimal point; for example, 2.67 meters = 267 centimeters.

The use of prefixes to indicate the factors of 1000, 0.001, and so on, is quite useful. For example, the prefix *centi* means $\frac{1}{100}$ or 10^{-2} of whatever it is attached to, as 1 centimeter = 0.01 meter. Those we will need to know are as follows:

mega — $10^{6} = 1,000,000$ kilo — $10^{3} = 1,000$ centi — $10^{-2} = 0.01$ milli — $10^{-3} = 0.001$ micro — $10^{-6} = 0.000001$

For example, one millimeter $= 10^{-3}$ meter, and is a convenient unit for small distances. The preceding list is extended considerably in both directions for use elsewhere in the subject of physics, but we do not need more than the above.

To save space in diagrams and in mathematical equations, it is convenient to abbreviate the names of the units in the recommended fashion, as follows:

meter — m centimeter — cm millimeter — mm kilometer — km foot — ft

Abbreviations for names of other quantities will be given as they are defined.

The concept of length is involved in what we call a *displacement*, which we obtain when we move an object from one place to another. Physicists cannot talk for very long without drawing pictures and this is a good place to start; Fig. 1 illustrates the displacement of an object moved from point A to point B. The amount of the displacement obviously will be measured in terms of some unit of length. The physicist, who never uses a short word when a long one will do, calls the amount of a displacement its *magnitude*. A displacement is not specified completely if only its magnitude is given; it is also necessary to give its direction. We will meet with a number of physical quantities of this sort, which involve both direction and magnitude; they are called *vector* quantities. In Fig. 1 the arrow drawn from A to B can be



FIG. 1. A displacement has both magnitude and direction.

used to represent the displacement, both in magnitude and direction, and this representation will be used with other vector quantities as needed. Most of the time we will not be concerned with the directions of displacements and other vector quantities, but only with their sense —that is, whether they are positive or negative.

After we have defined the fundamental units, we can combine them in various ways and obtain new ones that are called *derived* units. From the unit of length we can obtain the unit of area as a derived unit. The area of a rectangle, for example, is obtained by multiplying

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its length by its width; a square 5 meters on a side would have an area 5 meters \times 5 meters = 25 square meters. The square meter is a derived unit. Since it is obtained by multiplying meters by meters, which is equivalent to meters squared, it is convenient to use m² as an abbreviation when needed. Similarly, the volume of a box, found by multiplying the area of its base by its height, would be expressed in cubic meters, abbreviated m³. Many of the derived units we shall obtain subsequently will be used often enough to be given special names. For example, the *acre* is a unit of area in the English system.

Time

The next fundamental unit chosen is that of *time*. As with length, we have an intuitive feeling for what we mean by time; what is needed for quantitative purposes is a defined unit in which to measure it. For a considerable period of history, the rotation of the earth served as a convenient basis. From observations of the sun moving across the sky, taken over a considerable period of time, we can work out an average *solar day*. This day is divided into 24 hours (abbreviated hr), each hour into 60 minutes (min), and each minute into 60 seconds (sec). This means that there are 86,400 seconds in a day, so we may define the second as 1/86,400 of a solar day. Recently it has been found that the earth wobbles a bit as it rotates on its axis and by present-day standards is not really a good clock; as a result, it has become necessary to define the second in other terms. These problems do not concern us, and for our purposes it is sufficient to consider the second as something we can read from the second hand on a watch.

Velocity, Speed, and Acceleration

From the fundamental units of length and time, we may now obtain some important derived units that we will need subsequently.

The first of these is again one with which we are intuitively familiar; it concerns objects in motion. For such an object, its displacement, as measured from some starting point as a reference, is continually changing. The rate at which the object's displacement is changing is called its *velocity*. Since displacement in general involves both a direction and a magnitude, so also does velocity. To describe an object in motion, therefore, we must state not only how fast it is moving, but in what direction; velocity is another vector quantity.

For our purposes, we will generally not be concerned with the directions in which things are moving, but only with the sense of the motion —up or down, for example. We may then simplify matters by considering only the magnitude of the velocity; this is a quantity we call *speed*, and one with which we are quite familiar in this automotive age.

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We will measure the speed of an object by observing how many units of distance are covered in one unit of time. For example, we may observe an automobile moving along the highway and find that it travels, say, 135 meters in a time interval of 5 seconds. Its speed is then 135 meters per 5 seconds, or 27 meters per second (abbreviated m/sec). Both the amount—27, in this case—and the unit—meters per second—are equally important in describing the speed. In general, if an object moves a distance D meters in a time t seconds, its speed S in meters per second will be

 $S = \frac{D}{t}$ (1)

We could equally well measure the speed of the car in English units, in which case we would find it to be 88 feet per second. A person in the car could look at its speedometer and find its speed to be 60 miles per hour. All of these descriptions of the speed are equivalent. In fact, a sailor riding in the car would be correct in saying that its speed was 52 knots, the term *knot* being applied to a unit of speed of 1 nautical mile per hour. (Sometimes one hears the redundant term "knots per hour"; the person using it may be a sailor, but he is not a physicist.)

If the speed of an object is given, we may turn the above formula around and write

$$D = S \cdot t, \tag{2}$$

so that if we know the speed of an object and the time during which it moves, we can find the distance it covers. Obviously we must use consistent units; if the speed is in miles per hour, the time must be in hours, not seconds.

Both the above simple formulas are based on the assumption that the object is moving with a constant or uniform speed; if this is not so, they will not give correct answers. The corresponding mathematical formulas for the case of nonuniform speed are more complicated, and we will not need them.

Nevertheless, the fact that the speed of an object in motion need not be constant introduces us to an important new concept. Just as velocity is the rate at which the displacement of an object is changing, so the rate at which its velocity is changing is a quantity we call *acceleration*. Since velocity involves a direction, so also does acceleration, making it another vector quantity.

The velocity of an object may change either in direction, in magnitude, or in both; in any case, there is acceleration. For example, the object may be moving in a circle with a constant speed; the direction of its velocity is then constantly changing, and it is being accelerated. However, in the examples we will be considering, motions will gen-

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erally be in straight lines, so no change in direction will be involved. Hence, for our purposes, we may consider acceleration as the rate of change of speed. If the object is speeding up, the acceleration is positive—that is, in the same direction as the object is moving. If the object is slowing down, the acceleration is negative, in the opposite sense. A negative acceleration is sometimes called a *deceleration*.

To illustrate, let us again consider the car on the road. If we push down on the pedal supplying gasoline to the motor, the car speeds up, so this pedal is sometimes called the "accelerator." If we step on the brakes, the car slows down, so we could call the brakes "decelerators." Suppose the car is speeding up, and at a particular instant the speedometer (calibrated for our purposes in meters per second instead of miles per hour) reads 10 meters per second. Now assume that the speed of the car increases from 10 meters per second to 30 meters per second in 5 seconds; this is a net change in speed of 20 meters per second, so we get for the acceleration, expressed as the rate at which the speed is increasing,

acceleration
$$=$$
 $\frac{20 \text{ m/sec}}{5 \text{ sec}} = \frac{4 \text{ m/sec}}{1 \text{ sec}}$

meaning that the speed increases by 4 meters per second each second. However, since algebraically

$$\frac{a/b}{c} = \frac{a}{bc},\tag{3}$$

we may write

acceleration $= \frac{4 \text{ m}}{\sec \cdot \sec}$.

Also, algebraically,

$$\sec \cdot \sec = \sec^2$$
,

so we may use this notation and write more concisely

acceleration
$$= 4 \text{ m/sec}^2$$
.

The unit of acceleration in the metric system is then 1 m/sec², read as "one meter per second squared."

Mass

The third fundamental physical quantity is not as easily defined as the first two, since it is not as intuitively evident. It is called *mass*. For our purposes it will be sufficient to say that mass is a property possessed by all matter. This is not a very complete definition, but we will hope

that the concept will become clearer in subsequent discussions. The mass of a given piece of matter is the same wherever it may bewhether on the surface of the earth or in outer space.

As with the other units, we measure masses by comparing them with a standard. The present standard of mass is the *kilogram* (abbreviated kg), which is defined as the mass of a particular cylinder of platinum kept along with the standard meter at the International Bureau of Weights and Measures. Replicas of the standard kilogram are used throughout the world as subsidiary standards.

The mass of any object can be compared to the standard mass by a process called *weighing*. We will discuss this process after we have unscrambled the concept of mass from another physical quantity that is intimately associated with it.

Force

This new quantity is again one with which we are quite familiar. It is called *force*, and is simply a push or pull. Practically everything we do in our everyday lives requires the application of forces: lifting objects, opening doors, or even simply standing. We recognize the existence of a force when it acts on our person by the physiological feeling of pushing or pulling that it produces.

In the external world, removed from any physiological sensation, a force can manifest itself in one of two ways. First, a force can produce a *distortion* in a piece of matter; that is, a force can alter the shape of an object. If we pull on a wire spring, we can stretch it out of its usual shape; if we push on a lump of putty, we can squash it into a different form. Hence, if we see a spring being stretched, or a lump of putty altering its shape, we may expect to find forces acting. The spring, if it is not stretched too far, will return to its original shape if the force is removed. Materials possessing this property are said to be *elastic*. Most substances possess this property to some degree; however, any of them will become permanently deformed if stretched too far.

Second, a force applied to a mass can produce an acceleration, setting it into motion if it is at rest or changing its motion if it is not. When we throw a ball, we certainly exert a force on it, and we observe that it accelerates until we release it. Hence if we observe a ball undergoing an acceleration, we may expect to find that a force is acting on it.

Like acceleration, a force has a direction associated with it; this was already implied somewhat in our statement that a force is a push or pull. A force exerted on a given mass may be up, down, or in any given direction; however, the acceleration produced will always be in the direction of the force. This seems fairly obvious, and can be checked in the laboratory; the world would be a strange place if this were not so.

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There is a force which we experience constantly and which acts on every object in the vicinity of the earth; this is the force of gravity. A famous physical law, the law of gravity, states (in part) that every piece of matter in the universe exerts an attractive force on every other piece of matter. Hence the large mass of the earth exerts this force on all objects near it; the force is directed toward the geometrical center of the earth, which means that for us on the surface of the earth the force is directed vertically downward. The force of gravity acting on a mass is given a special name; it is called its *weight*. This is illustrated in Fig. 2(a); the force is represented by an arrow whose direction is





that of the force and whose length is proportional to the magnitude of the force, as was done with other vector quantities.

Since every object on the earth has weight and since weight, being a force, can produce acceleration, we might expect that every object on the earth should be accelerated downward. This is sometimes the case, as we find when we accidentally drop something, but many objects around us are not accelerating at all. This is because the weight of the object may not be the only force acting on it. Suppose we tie a string to the mass of Fig. 2(a) and hold on to the string so the mass is at rest, as in Fig. 2(b). The weight of the mass is still there, and can be felt; in fact, we are pulling up on the string with a force equal to the weight. There are then two forces acting on the mass; they are equal in magnitude, but opposite in direction. Since the mass can not accelerate in two directions at once, it simply does not accelerate at all. The mass is then said to be in *equilibrium*. The two forces acting have balanced or canceled each other, so the net force acting is zero.

We can see that the equilibrium situation is a very common one. A block resting on the table, as in Fig. 3(a), is in equilibrium; the



FIG. 3. (a) A block in equilibrium on a table. (b) Equilibrium with four forces.

weight of the block pulling it down is canceled by the force of the table pushing it up.

According to another physical law, forces always occur in pairs; for any force, there is another one somewhere that has the same magnitude and is along the same line, but is acting in the opposite direction. In Fig. 2(b), we pull up on the string to support the mass; the string pulls down on us with an equal force. In Fig. 3(a) the earth pulls down on the block, and the block pulls up on the center of the earth with an equal force. The table pushes up on the block, and the block pushes down on the table with an equal force. In any given situation, however, we are concerned with only one of the forces of each pair; in Figs. 2(b) and 3(a), we are interested only in the forces exerted on the mass and on the block; we are not concerned with the forces they exert on their surroundings.

To return to the block on the table: Suppose we now push on the block in a horizontal direction. If the table were perfectly smooth, the block would accelerate along the table. However, most tables are not perfectly smooth, and if we do not push too hard we find that the block does not move. This is because a new force appears as soon as we start to push; this force is called *friction*.

The forces due to friction are curious things. For one thing, they are always in opposition; whatever the direction the block is pushed, the frictional force pushes in the opposite direction. For another, frictional forces can adjust themselves to any magnitude within limits, so that as we push more or less hard on the block, the friction becomes whatever is needed to cancel the push. The net force on the block, both horizontally and vertically, is then zero, so the block is still in equilibrium. Fig. 3(b) illustrates this equilibrium situation, with the block at rest.

However, there is generally a limit to the amount of the force that friction can develop, so if we push hard enough, we can get the block to accelerate. If the push is adjusted to just the right value, we can get the block to slide at a constant speed, without acceleration. Here the frictional force is still equal to the pushing force, and we still have equilibrium. The criterion for equilibrium is not that the block be at rest, but that it not be accelerating. Fig. 3(b) also illustrates this equilibrium situation, the block moving with constant speed.

Forces do not have to be directed only horizontally or vertically, but can be directed at other angles; we will then have more complicated equilibrium situations. For example, consider a mass hung by two strings, as shown in Fig. 4. The strings exert forces F_1 and F_2 , which must be directed along the strings. Then the two forces F_1 and F_2 are equivalent in all respects to a single force R, called the *resultant*, found by a simple rule: draw a parallelogram with F_1 and F_2 as two adjacent sides; the resultant force R, in magnitude and direction, is then given by the diagonal of the parallelogram as shown in Fig. 4. The resultant R must be equal and opposite to the weight of the mass in order for the system to be in equilibrium. Any two forces can be replaced by their resultant, as found by this method, without affecting the behavior of the system in which the forces are acting. We will find this principle useful later, as for example when we discuss the violin.

So far we have said nothing about how we are to measure the force or what our unit of force is to be. Since a force can produce either an



FIG. 4. Mass in equilibrium supported by two strings, illustrating the parallelogram rule. acceleration of an object or a distortion of its shape, we may in principle use either of these effects as a basis of measurement. The measurement of a force by using the distortion of an object will involve the elasticity of its material; this is an undesirable complication, particularly in establishing a unit which is to be used as a standard. To avoid this difficulty, forces are basically defined and measured in terms of the accelerations they produce.

To do this, we need the help of experiments in the laboratory. We assume that we already have some means of measuring forces, so that we can apply known forces to known masses and measure the resulting accelerations. If we apply various forces to a given mass, we find a quite reasonable result: the acceleration is directly proportional to the force and in the same direction, so that doubling the force, for example, will double the acceleration. Alternatively, if we apply a given force to different masses, we find that the acceleration is inversely proportional to the mass, so that doubling the mass will give half the acceleration. If we let M be the mass, F the magnitude of the force, and a the resulting acceleration, we can combine the above observations in the single equation

$$F = M \cdot a. \tag{4}$$

This equation is one of the fundamental equations of physics. It summarizes in one concise statement the results of centuries of thinking and speculating about the way things move. We will use it subsequently not so much in its mathematical formulation but rather for the insight it gives us into the behavior of things. It tells us that increasing the forces acting on a system will generally produce larger accelerations and hence larger velocities and displacements; increasing the masses of a system will have the opposite effect.

Eq. (4) also defines the unit of force. If we make M = 1 kilogram and a = 1 meter per second squared, the force F will be 1 kilogram meter per second squared, which is now the unit of force. Since we are going to use this unit quite often, we give it a special name and call it one *newton* (abbreviated N). The newton is then that force that imparts to a mass of one kilogram an acceleration of one meter per second squared. The unit of force is now defined in terms of the three fundamental units of length, mass, and time.

A simple application of Eq. (4) above is to freely falling bodies. For example, if the string supporting the mass in Fig. 2(b) is cut, the weight of the mass (the force of gravity on it) will cause it to accelerate downward. This will be true for any unsupported object, for which the only force acting is its weight. If we measure the acceleration of a falling body, it turns out to be 9.8 meters per second squared regardless of the mass of the body. (This will not ordinarily be true for a light object like a feather, since air friction supplies some supporting force. In a vacuum a feather will fall as fast as any other object.) For the falling body, the F in Eq. (4) is the weight of the body. It then follows from Eq. (4) that the weight of a body is proportional to its mass. Quantitatively, its weight in newtons is 9.8 times its mass in kilograms.

If a mass is accelerated by a force and the force removed when a given speed is reached, the mass will continue to move in a straight line with that speed. Only the application of another force can change the speed; in particular, the mass can be stopped only by applying a force in the direction opposite to its motion. This property of a mass in motion to remain in motion is called *momentum*.

The definition of force in terms of mass and acceleration is quite fundamental in that it does not depend on any material property other than mass. However, in practical work the use of accelerations to measure forces is rather inconvenient, so instead we may use for this purpose the distortions forces produce in material objects. For example, a coiled spring made of some elastic wire such as spring steel may be stretched by applying forces to it, and the amount of stretch is proportional to the force, provided the spring is not stretched so far as to acquire a permanent deformation. (Many other systems besides springs have this property, which will turn out to be of great musical importance.) If we hang the spring from a support, as in Fig. 5(a), the bottom of the spring will be in a certain position, which we may



FIG. 5. (a) A spring balance. (b) Weighing a mass by means of a spring balance.

mark as "0" on the support. A force of one newton downward will stretch the spring to a new point which we may mark "1" on the support. A force of two newtons will then stretch it twice as far, giving point "2," and so on. This arrangement is called a *spring balance*, and is useful for practical measurements of forces.

A spring balance can be used to measure masses, since the weight of an object is proportional to its mass. If a mass is suspended from the spring as in Fig. 5(b), a position can be found for which the mass remains at rest, with the spring stretched a certain amount. This is obviously an equilibrium position, for which the weight of the mass downward is balanced by the force of the spring upward. Masses can thus be compared by the amount of stretch they produce in the spring; this is one method of weighing, which we mentioned earlier.

This method of weighing has the disadvantage of depending on the elasticity of the spring. Another method of weighing is by the use of a *beam balance*, or platform balance, as illustrated in Fig. 6. This device



FIG. 6. A beam balance for comparing masses.

consists of two pans suspended from a beam which is supported at its center by means of a pivot. The beam will tip to one side or the other depending on which pan has the larger downward force acting on it; if the forces are equal, the beam remains horizontal. Since equal masses will have equal weights, the beam balance thus provides a method for determining when masses are equal. Masses can thus be weighed with this arrangement by comparing them directly to a standard.

On the moon, the force of gravity is about one-sixth what it is on the earth. A spring balance marked off to read weights on the surface of the earth would give quite erroneous readings if taken to the moon; its reading for a given mass would be one-sixth as much. A beam balance, on the other hand, would read correctly, since it compares masses directly.

In our discussion up to this point we have avoided the use of English units. The reason for this is that the common term pound (abbreviated lb) is used in our everyday lives as both a unit of mass and a unit of force, with no distinction between the two. Ordinarily this causes us no trouble, since a mass of one pound, for example, has a weight of one pound. However, if we wish to use these quantities in physical formulas, we must be careful to distinguish between pounds force and pounds mass. For example, an object falls with an acceleration of 32 feet per second squared, this being the acceleration produced by a force of one pound acting on a mass of one pound. Hence Eq. (4) above obviously does not work with these units. Furthermore, a man with a mass of 150 pounds and having a weight of 150 pounds on the earth would have the same mass on the moon, but his weight would go down to 25 pounds. In a satellite orbiting the earth, his mass would still be 150 pounds, but his weight would be zero. This confusion of terms makes it desirable to avoid the English system as much as possible.

Pressure

In our discussion of forces we have so far considered that they act essentially at points. The string supporting the mass of Fig. 2(b) is acting at a single point. The weight of the mass may also be considered to act at a point; if the mass is a sphere, this point is at its center. Similarly in the other examples, there is no need to think of the forces as acting at anything but points.

However, there are other situations in which a force is definitely not acting at a point, but is instead spread out over an area. In this case we can think of the amount of force that is acting on each unit area; this is a quantity called *pressure*, one which we will refer to very often. The unit of pressure will be newtons per square meter (abbreviated N/m^2). If a force F acts over an area A, the pressure p is given simply by

$$p = \frac{F}{A} \,. \tag{5}$$

Fluids exert pressures on their containers, for example, and anything immersed in them. Fig. 7 illustrates forces due to pressure acting on the surface of a vessel containing liquid, and on the surfaces of an object immersed in the liquid. The forces due to pressure are always perpendicular to the surface on which they act.

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We may illustrate the concept of pressure further with a simple example. Suppose Fig. 7 represents a square tank whose bottom measures 0.5 meters \times 0.5 meters and which contains water to a depth of 2 meters. The total mass of water in the tank will then be 500 kilograms. Since each kilogram weighs 9.8 newtons, the total weight of water will



FIG. 7. Forces exerted by the pressure of a liquid on the container and on an immersed object.

be $500 \times 9.8 = 4.9 \times 10^3$ newtons. This force is spread over an area of 0.25 square meters, so the pressure p on the bottom of the tank, as given by Eq. (5), is

$$p = \frac{4.9 \times 10^3 \text{ N}}{0.25 \text{ m}^2} = 1.96 \times 10^4 \text{ N/m}^2.$$

The same pressure is exerted on the side of the tank at the bottom, as indicated by the horizontal arrow near the bottom of the tank in Fig. 7. The pressure at a point in a liquid is produced only by the amount of liquid above that point; hence at a point higher up along the side of the tank, the pressure is less. Halfway up the side, the pressure will be half that at the bottom, and at the top water surface, it will be zero. This decrease in pressure at points higher up the side of the tank is illustrated by the horizontal arrows in Fig. 7.

In English units, pressure is usually expressed in pounds force per square inch. When we ask the service station attendant to put 30 pounds in our tires, we do not mean that we want a mass of 30 pounds of air in them. What we actually want is enough air to give a pressure of 30 pounds force per square inch on the inside of the tire.

We spend our lives surrounded by the earth's atmosphere, which exerts a pressure on everything in it, like any other fluid. At sea level, this pressure amounts to very nearly 10⁵ newtons per square meter, or (in English units) about 15 pounds force per square inch. It decreases with altitude; at an elevation of 15,000 feet is about half that at sea level. Normally we are unaware of this pressure, but we do notice changes in it if they are large enough, as when we go up in an airplane or under water; generally the change in pressure manifests itself in our ears as curious noises or perhaps pain. The actual value of the atmospheric pressure at any given place changes a little from time to time; its value at any given time is called the *ambient pressure*. Slow changes in the ambient pressure are of interest to meteorologists, furnishing information on changes in the weather. Small but rapid changes in the ambient pressure produce sensations in the ear which we call *sound*, and which we will subsequently study in considerable detail.

Work and Energy

The terms *work* and *energy* have various shades of meaning in our everyday lives. In physics, however, they have very specific meanings. If a force acts on an object and results in the object moving, then physical *work* is done by the force. For example, in Fig. 3(a), neither the force of gravity nor the force of the table acting on the block does work, since the block does not move. In Fig 3(b), however, if we push hard enough on the block to move it, we will do work.

The work that is done is measured by the product of the force times the distance moved in the direction of the force. If F represents the force and D the distance moved, then the work W done by the force is

 $W = F \cdot D$

We could express this in newton meters, but since in physics we use this quantity a great deal, we give the unit of work a name of its own and call it a *joule* (abbreviated J). If a force of one newton acts through a distance of one meter, it does one joule of work.

The distance used in Eq. (6) above must be that moved in the direction of the force. In Fig. 3(b), for example, the force of gravity on the block does no work when the block moves along the table because there is no motion in the direction of this force.

In order to do work, we must expend *energy*. Any system which can do work must have a supply of energy available from somewhere to accomplish this work. This is a consequence of a very fundamental physical principle known as the law of *conservation of energy*, which states that energy cannot be created or destroyed, but can only be transformed from one form into another. To the physicist, this makes energy just as real as money; it cannot be spent unless there is a supply somewhere to draw on, and once spent it is generally gone forever. In our everyday lives we are becoming increasingly conscious of the fact that the energy we need so much of cannot be created from nothing, and where it will come from in the future is becoming a matter of considerable concern.

The definition of physical work given above is somewhat at variance with our intuitive feelings about things. If I lift a heavy weight off the

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floor to a certain height and hold it there, I am doing physical work while I lift it, but not while I am holding it steadily aloft. However, everyone knows that holding a heavy object up off the floor is "hard work," even if the object is kept motionless. It is true that no physical work is being done in this case; however, the human body is a very complex system, and it turns out that muscles have to use up energy just to exert a force, so physiological work must be done in order to hold a heavy object off the floor. A table, however, can hold up the object indefinitely without expending energy and getting tired. Consequently, we will disregard the physiological complications and adopt the definition of work as given here.

Energy occurs in various forms. The kind we are discussing here in moving things around is *mechanical* energy. It occurs in two different forms; *potential* energy, which is the energy a system can have because of its configuration, and *kinetic* energy, which is energy a system can have because of its motion.

To illustrate: If we lift a mass of one kilogram off the floor to a height of two meters, we will have to exert a force of 9.8 newtons, and so will do 19.6 joules of work, as given by Eq. (6) above. The mass can now exert forces on other objects by reason of its weight, and so can do work in turn; the amount that could be obtained is just what was given it, namely 19.6 joules. The energy that the mass has because of its elevation off the floor is thus potential energy.

If we take the same mass and throw it, we cause it to accelerate by exerting a force on it, and so do work. If we exert a force of 9.8 newtons to throw it, and apply this force over a distance of two meters, we will again do 19.6 joules of work. Because of its momentum, the mass can exert forces on other objects, as by striking them, and so can do work on them in turn. As before, the amount that could be obtained is just what was given it, again 19.6 joules. The energy the mass has because of its motion is thus kinetic energy.

These two kinds of energy are interchangeable. If we raise the mass off the floor and let go of it, it falls. As it descends it loses potential energy but, since it speeds up, it gains kinetic energy. When it strikes the floor it has an amount of kinetic energy just equal to the amount of potential energy it had when released. This is one example of the law of conservation of energy, in this case, mechanical energy; in this illustration the total energy of the falling mass, kinetic plus potential, remains constant at 19.6 joules as the mass falls.

Another form of energy with which we are all familiar is *heat*, which is what we put into things to make them warmer. The law of conservation of energy applies also to heat. When the mass discussed in the previous paragraph strikes the floor, it stops, and its mechanical energy disappears. However, collision of the mass with the floor generates heat, and the heat energy produced is equal to the mechanical energy that disappears, namely 19.6 joules.

Whenever forces do mechanical work in overcoming friction, heat is generated. When we push the block in Fig. 3(b) across the table, we do work overcoming the friction of the block sliding on the table. The mechanical energy expended then appears as heat produced in the sliding surfaces. It is the fate of all mechanical energy to eventually disappear into heat energy (except for small amounts that may be transformed into other forms of energy we have not discussed). When this happens, the energy is for practical purposes mostly gone, like money spent. The reverse process of getting mechanical energy from heat is possible, but unfortunately only to a limited extent; a large part of the heat energy in the world around us cannot be utilized as a source of useful work.

Power

The concept of work developed in the previous section leads us to another concept which is equally important: the rate at which work is done. When we raise the mass off the floor in the illustration above, we may do so quickly or slowly. The work of 19.6 joules done will be the same in either case. If we did it in two seconds, we would do work at the rate of 9.8 joules per second. If we required 20 seconds, we would do work at the rate of 0.98 joules per second. The rate at which work is done is called *power*. It could be measured in joules per second; however, we use the unit of power so much that we give it a special name, the *watt* (abbreviated W). An expenditure of energy at the rate of one joule per second is a power expenditure of one watt.

This unit is somewhat familiar to us since our electrical equipment is rated according to the number of watts it uses when operating. A 100-watt light bulb uses up 100 joules each second it is burning. Ten such lights would use 1000 watts, which is one *kilowatt*. To burn these ten lights for 24 hours would require $1000 \times 24 \times 60 \times 60 = 8.64 \times$ 10^7 joules. The local electric company is in the business of selling electrical energy and bills us at the end of the month according to how much energy we have used. The company does not use the joule as a unit of energy, but instead uses what is called a *kilowatt-hour*, the amount of work done by one kilowatt expended for one hour; this amounts to $1000 \times 3600 = 3.6 \times 10^6$ joules. The ten lights above would then use 24 kilowatt-hours in the 24 hours of burning. At a few cents per kilowatt-hour, the joules the electric company sells us are quite inexpensive.